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Neutrino masses in the R_p violating MSSM

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Abstract: We compute one loop neutrino masses in the R-parity violating Minimal Supersymmetric Model, including the bilinear $\not{R}_p$ masses in the mass insertion approximation. To the order we calculate, our results are independent of the Higgs-lepton basis choice. We have a variety of perturbative parameters—gauge, Yukawa and trilinear couplings, and R_p violating masses. Their relative magnitudes determine which diagrams are relevant for neutrino mass calculations. We find new loop diagrams which can be relevant and have frequently been neglected in the past. For the Grossman-Haber neutral loop contribution to the neutrino mass matrix we obtain explicit analytic results.

Keywords: Solar and Atmospheric Neutrinos, Supersymmetric Standard Model, Neutrino Physics.

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Contents

1. Introduction

An interesting challenge in particle physics is to construct a consistent model which can accomodate neutrino oscillations in order to explain atmospheric and solar neutrino data [1]. Considering the results of SuperKamiokande and all solar neutrinos experiments, at least two different values of Δm^2 and $\sin^2 2\theta$ are necessary to account for the data. Neutrino masses cannot be generated in the Standard Model (SM), so it is necessary to consider extensions of the SM [2]. An interesting possibility for generating neutrino masses are models in which lepton number violation occurs. In these models Majorana neutrino masses are generated by interactions which violate lepton number in two units: $\Delta L = 2$.

Among the possible models we consider the Minimal Supersymmetric Standard Model without imposing R-parity. The quantum number R is defined as $R_p = (-1)^{L+3B+2S}$, where L, B, S are the lepton and baryon number and the spin of the particle, respectively [3]. Imposing R-parity in a supersymmetric model has the advantage of suppressing simultaneously the presence of baryon and lepton number violating interactions, which are strictly constrained from proton decay experiments and other low-energy processes [4]. However, one can impose a less stringent constraint in which all interactions conserve baryon number. Thus only lepton number violating terms are present in the lagrangian. These terms allow left-handed neutrinos to obtain a Majorana mass, at tree level through mixing with the neutralinos, and through loop diagrams that violate lepton number (in two units) [5, 6, 7].

In the SM, the Higgs and leptons have the same gauge quantum numbers. However, they cannot mix because the Higgs is a boson and the leptons are fermions. In a supersymmetric model this distinction is removed, so the down-type Higgs and sleptons can be assembled in a vector $L_J = (H_d, L_i)$ with $J : 4 \dots 1$. With this notation, the superpotential for the supersymmetric SM with R_p violation can be written as

$$W = \mu^J H_u L_J + \lambda^{JK\ell} L_J L_K E^c_\ell + \lambda'^{Jpq} L_J Q_p D^c_q + h_t^{pq} H_u Q_p U^c_q \,. \tag{1.1}$$

The R_p violating and conserving coupling constants have been assembled into vectors and matrices in L_J space: we call the usual μ parameter μ_4, and identify the usual $\epsilon_i = \mu_i$, $\frac{1}{2}h_e^{jk} = \lambda^{4jk}$, and $h_d^{pq} = \lambda'^{4pq}$. Lower case roman indices i, j, k and p, q are lepton and quark generation indices. We frequently suppress the capitalised indices, writing $\vec{\mu} = (\mu_4, \mu_3, \mu_2, \mu_1)$.

We also include possible R_p violating couplings among the soft SUSY breaking parameters, which can be written as

$$\begin{aligned} V_{soft} = {} & \frac{\tilde{m}_u^2}{2} H_u^\dagger H_u + \frac{1}{2} L^{J\dagger} [\tilde{m}_L^2]_{JK} L^K + B^J H_u L_J + \\ & + A^{ups} H_u Q_p U^c_s + A^{Jps} L_J Q_p D^c_s + A^{JKl} L_J L_K E^c_l + \text{h.c.} \end{aligned} \tag{1.2}$$

Note that we have absorbed the superpotential parameters into the A and B terms; e.g. we write $B^4 H_u H_d$ not $B^4 \mu^4 H_u H_d$.[1] We abusively use capitals for superfields (as in (1.1)) and for their scalar components.

We have put the Higgs H_d into a vector with the sleptons, and combined the R_p -violating with the R_p conserving couplings, because the lepton number violation can be moved around the lagrangian by judiciously choosing which linear combination of hypercharge $= -1$ doublets to identify as the Higgs/higgsino, with the remaining doublets being sleptons/leptons. However, this freedom to redefine what violates L is deceptive, because phenomenologically we know that the leptons are the mass eigenstate e, μ and τ , so we know what lepton number violation is. Lepton number is defined in the charged lepton mass eigenstate basis — the freedom to choose which direction is the Higgs in the lagrangian just means that there is not a unique interaction eigenstate basis. There are two possible approaches to this fictitious freedom; choose to work in a lagrangian basis that corresponds to the mass eigenstate basis of the leptons, or construct combinations of coupling constants that are independent of the basis choice to parametrise the R_p violation in the lagrangian [6, 8, 9]. These invariant measures of R_p violation in the lagrangian are analogous to Jarlskog invariants which parametrise CP violation.

The standard option is to work in a basis that corresponds approximately to the mass eigenstate basis of the leptons. For instance, if one chooses the Higgs direction

[1] We do this because B_J is a vector — a one index object — in $\{L_J\}$ space. From this perspective, giving it two indices can lead to confusion.

in L_J space to be parallel to μ_J, then the additional bilinears in the superpotential μ_i will be zero. In this basis, the sneutrino vevs are constrained to be small by the neutrino masses, so this is approximately the lepton mass eigenstate basis. Lepton number violation among the fermion tree-level masses in this basis is small by construction, so it makes sense to neglect the bilinear R_p violation in setting constraints on the trilinears, as is commonly done (for a review, see e.g. [4]. For a careful analysis including the bilinears, see [10]. Note that in many cases, the most stringent constraints on the additional $R-$parity violating parameters come from neutrino anomaly data [11].)

The aim of the "basis-independent" approach is to construct combinations of coupling constants that are invariant under rotations in L_I space, in terms of which one can express physical observables. By judiciously combining coupling constants one can find "invariants" which are zero if R_p is conserved, so these invariants parametrise R_p violation in a basis-independent way. For instance, consider the superpotential of equation (1.1) in the one generation limit, $I : 4 \ldots 3$. It appears to have two R_p violating interactions: $\mu_3 H_u L$ and $\lambda' LQD^c$. It is well known that one of these can be rotated into the other by mixing H_d and L [5]. If

$$\begin{aligned} H_d' &= \frac{\mu_4}{\sqrt{\mu_4^2+\mu_3^2}} H_d + \frac{\mu_3}{\sqrt{\mu_4^2+\mu_3^2}} L \\ L' &= \frac{\mu_3}{\sqrt{\mu_4^2+\mu_3^2}} H_d - \frac{\mu_4}{\sqrt{\mu_4^2+\mu_3^2}} L\,, \end{aligned} \tag{1.3}$$

then the lagrangian expressed in terms of H_d' and L' contains no $H_u L'$ term. One could instead dispose of the $\lambda' LQD^c$ term. The coupling constant combination that is invariant under basis redefinitions in (H_d, L) space, zero if R parity is conserved, and non-zero if it is not is $\mu_4 \lambda' - h_d \mu_3 = (\mu_4, \mu_3) \wedge (h_d, \lambda')$. For a more detailed discussion of the approach followed in this paper see e.g. [8, 9, 12].

Basis-independent coupling constant combinations which parametrise the amount of $\not{R}_p$ can be constructed in various ways. Their advantage over the coupling constants is that they *cannot* be set to zero by a basis transformation. So we define the following invariants $\{\delta\}$ which reduce to the coupling constant of the same indices in the basis where the sneutrinos do not have vevs (the $\rightarrow$ is to what the δ becomes in this basis).

$$\begin{aligned} \delta_\mu^i &= \frac{\vec{\mu} \cdot \lambda^i \cdot \vec{v}}{|\vec{\mu}| m_i^e} \rightarrow \frac{\mu_i}{\mu_4} \\ \delta_{\lambda'}^{ipq} &= \frac{\vec{\lambda}'^{pq} \cdot \lambda^i \cdot \vec{v}}{m_i^e} \rightarrow \lambda'^{ipq} \\ \delta_B^i &= \frac{\vec{B} \cdot \lambda^i \cdot \vec{v}}{|\vec{B}| m_i^e} \rightarrow \frac{B_i}{B_4} \\ \delta_\lambda^{ijk} &= \frac{\vec{v} \cdot \lambda^i \lambda^k \lambda^j \cdot \vec{v}}{m_i^e m_j^e} \rightarrow \lambda^{ijk} \end{aligned} \tag{1.4}$$

We have chosen the Q and D^c bases to make $\vec{v} \cdot \lambda'^{pq}$ diagonal — that is, to diagonalise the down-type quark mass matrix. We assume that this is also the mass eigenstate basis for the squarks. Similarly, we choose the E^c basis to make $\vec{v} \cdot \lambda^i \lambda^k \cdot \vec{v} \propto \delta^{ik}$. Combined with our definition of the lepton directions as $\hat{L}_i = \vec{v} \cdot \lambda^i / m_i^e$, ($m_i^e = |\vec{v} \cdot \lambda^i|$) this means we diagonalise the charged lepton mass matrix induced by the Higgs vev. Here we are neglecting R-parity violating bilinears that mix the charginos and charged leptons; these masses will be included as mass insertions in perturbation theory.

The mass matrix of the neutralinos is composed by the 4 neutralino fields of the R-parity conservng MSSM and the 3 neutrino fields. In an interaction eigenstate basis they are

$$\chi_{\rm int} = (\tilde{B}, \tilde{W}_3, \tilde{h}_u, \tilde{h}_d, \nu_\tau, \nu_\mu, \nu_e)\,, \tag{1.5}$$

where as previously discussed, in $\not{R}_p$ models there is no unique interaction eigenstate choice for the basis in $\tilde{h}_d$ and lepton space. In this paper we define the basis where the sneutrinos do not have vacuum expectation values to be the interaction eigenstate basis. We number the elements of χ in *reverse* order, so $\chi^7_{\rm int} = \tilde{B}$, and $(\chi^4_{\rm int}, \chi^3_{\rm int}, \chi^2_{\rm int}, \chi^1_{\rm int}) = (\tilde{h}_d, \nu_\tau, \nu_\mu, \nu_e)$. We use this numbering so that (χ^3, χ^2, χ^1) will be the mass eigenstate neutrinos.

In the R_p conserving MSSM, lepton number is conserved, and the first four fermions have majorana masses, while the three neutrinos are massless. The mass matrix can be diagonalised with a matrix:

$$\begin{bmatrix} Z & 0 \\ 0 & I \end{bmatrix}, \tag{1.6}$$

where Z is a 4×4 matrix and I is the 3×3 identity matrix.

The purpose of this paper is to address and clarify the issue of different basis in $\not{R}_p$ neutrino mass calculations. We construct "basis-independent" estimates of each loop diagram contributing to the neutrino mass matrix. These estimates are in an arbitrary lagrangian basis. They lead us to include the bilinear $\not{R}_p$ masses in the mass insertion approximation. This introduces a new diagram, and new contributions from the usual diagrams. These new terms resolve the basis related puzzles that arise in $\not{R}_p$ neutrino mass calculations when the bilinears are neglected in the loops. Clearly neutrino masses should not depend on the basis they are computed in; however, if the $\not{R}_p$ masses are neglected in the loops, then what one is neglecting depends on the basis. We will return to this in the discussion at the end. Having verified the irrelevance of performing basis independent calculations, we present estimates and the calculation of the neutral loop in the basis where the sneutrinos do not have vacuum expectation values, because the results are more compact than the basis independent estimates.

In section 2 we introduce our procedure for obtaining neutrino masses, and in section 3 we estimate the basis-independent contributions from all loops. In section 4

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we provide complete analytic expressions for the neutral loop, and compare with the exact result previously obtained in the literature [12]. The latter calculation is done in the MSSM mass eigenstate basis. The neutral graph is of interest as it has been analysed very little in the literature and furthermore it can provide strong constraints on Higgs physics in this model. In section 5 we discuss in more detail the issues related to the basis choice and conclude.

2. Perturbation theory with $\not{R}_p$ masses

Neutrino masses are observationally known to be small, so we can compute them in perturbation theory. There are two ways to include the $\not{R}_p$ bilinears in one-loop calculations of neutrino masses. One approach is to diagonalise the tree level mass matrices for neutral and charged scalars and fermions, and calculate the one-loop contributions to the neutrino mass matrix using a tree-level mass eigenstate basis for the propagators in the loop [13]. This is usually done numerically. Note that for $\delta_\mu \neq 0$, $\delta_B \neq 0$, the charged and neutral Higgses (MSSM neutralinos) mix with the sleptons (leptons). The alternative, which we follow here, is to propagate particles in the MSSM mass eigenstate basis, and include the $\not{R}_p$ masses as mass insertions [14] in perturbation theory. For "sensible" basis choices that are close to the MSSM, the $\not{R}_p$ bilinear masses and couplings are small, so including them as mass insertions should be adequate.

The algorithm for computing neutrino masses, with $\not{R}_p$ bilinears included in the mass insertion approximation, consists of the following steps:

1. choose a basis where the R_p violating parameters are small, e.g. the basis where the sneutrinos have no vevs, or where there are no $\mu_i h_u \ell_i$ terms.

2. diagonalise the heavy neutralino 4×4 matrix.

3. calculate the tree-level seesaw neutrino mass.

4. calculate loop contributions to the 3×3 neutrino mass matrix, including the $\not{R}_p$ bilinear masses as mass insertions.

5. diagonalise the resulting 3×3 neutrino mass matrix.

It is well known that in the $\not{R}_p$ MSSM only one of the neutrinos acquires a mass at tree-level through the see-saw mechanism due to the mixing of the neutrinos and neutralinos. We will focus on the (finite) loop contributions to neutrinos which are massless at tree level. If the tree-level mass is non-zero, there are gauge loop corrections which must be renormalised. To avoid these, we for simplicity neglect loop corrections to the neutrino who is massive at tree level. We will address loops containing gauge bosons, and issues of renormalisation, in a subsequent publication.

If the tree-level mass is smaller than the loop contribution, it can be set to zero ($\delta_\mu = 0$), and one-loop masses can be calculated for three neutrinos.

We only consider one-loop diagrams contributing to the neutrino mass matrix — higher order gauge loops may be larger than one loop diagrams with Yukawas/trilinears at the vertices, but gauge couplings have no flavour indices so pure gauge loops will give a mass aligned with that of some lower order diagram. Two-loop diagrams with Higgs exchange rather than mass insertions on the internal lines should be suppressed by an additional factor of $1/(16\pi^2)$. We neglect the loop corrections to the neutralino sector and to neutrino-neutralino mixing. These would contribute to the two loop neutrino mass matrix. To compute the one-loop neutrino mass matrix, we propagate tree level mass eigenstates in the loop with tree level interactions.

In this paper we concentrate on step 4. We draw diagrams and make basis-independent estimates in an arbitrary lagrangian basis, which we assume satisfies step 1. The advantage of this is that the basis-independent coupling constant combination controlling the neutrino mass can be read off these diagrams. However, to get the correct dependence on MSSM masses and mixing angles, we should work in the MSSM mass eigenstate basis. We do this for the neutral loop in section 4, defining the Higgs to be the direction in L_I space that gets a vev. This basis choice in L_J space is close to the charged lepton mass eigenstate basis because the misalignment between $\vec{\mu}$ and $\vec{v}$ must be small, so μ_i in this basis are small. There are also no lepton number violating D-term masses proportional to the sneutrino vev. Since we are perturbing in lepton number violating masses, this reduces the number of interactions we must consider. This is therefore a good basis from which to do perturbation theory in $\not{R}_p$ parameters. We define our "MSSM mass eigenstate basis" (in an $\not{R}_p$ theory) by diagonalising the mass matrices with all the $\not{R}_p$ parameters set to zero, then reintroducing the $\not{R}_p$ parameters in the L_J basis where the sneutrino have no vevs.

We have many parameters in which we will be perturbing to compute loop neutrino masses: Standard Model Yukawa couplings h and gauge couplings g, of very different sizes, and bilinear and trilinear $\not{R}_p$ couplings of unknown size. We parametrise the size of the $\not{R}_p$ couplings in a basis-independent way via the parameters δ introduced earlier. We would like to order them in magnitude, so that we consistently include all contributions greater than a certain size.

It is clear that $h \ll g$, and that the Yukawas are known and of different sizes. We do not know the size of the various $\{\delta\}$; so we consider three possibilities:[2]

A Assume bilinear $\not{R}_p$ is small and can be neglected in the loops. This is the case if $h_e\delta_\lambda, h_d\delta_{\lambda'} \gg g\delta_B, g\delta_\mu$. One gets the usual loop diagrams of figures $1a$ and $1b$.

[2] We have suppressed the family indices for simplicity.

B Neglect δ_μ, but include the soft bilinear $R\!\!\!/_p$ terms (B_i and m^2_{4i}) in perturbation theory. This means $h_e\delta_\lambda, h_d\delta_{\lambda'} \sim g\delta_B$, but $g\delta_\mu \ll h_e\delta_\lambda, h_d\delta_{\lambda'}$. This gives the neutral Grossman-Haber loop of figure 1c, and additional contributions to the usual loop of figure 1a.

C Include all bilinear $R\!\!\!/_p$ terms in perturbation theory. There are additional contributions to diagrams 1a, 1b, and 1c, and a new diagram 1d.

Note that models with only bilinear $R\!\!\!/_p$ can fall into any of these three categories. A model where the misalignment between the vev and the Yukawa couplings is much greater than between the vev and $\vec{\mu}$ would fall into case A. Case C corresponds to a model where the misalignment between the vev $\vec{v}$ and $\vec{\mu}$ is similar to the misalignment between the Yukawa couplings and $\vec{v}$.

3. Estimates

The possible diagrams that contribute to the neutrino mass must contain two scalar-fermion-fermion couplings at the vertices of the loop, and two $\Delta L = 1$ interactions. The loop can contain coloured and colour-singlet charged or neutral particles, and we can have either two gauge couplings, one gauge and one Yukawa/trilinear coupling, or two Yukawa/trilinear couplings at the vertices.

The charged coloured loop is figure 1b. This has Yukawa or trilinear couplings at the vertices, where a higgsino or neutrino interacts with a quark and a squark. Lepton number violation can be due to the trilinears, or to mass insertions mixing the neutrino with a neutralino on the external legs.

In figure 1c we have the contribution from the neutral loop. At the vertices we must have two gauge couplings. It is not possible to have vertices with combinations of gauge-Yukawa and Yukawa-Yukawa couplings for a neutral loop, because the vertex with the Yukawa coupling would involve an E^c. This diagram has been previously considered in [7], and we will refer to it as the Grossman-Haber (GH) diagram. Lepton number violation, $\Delta L = 2$, occurs due to mass insertions on the internal scalar propagator. In the $\langle\tilde{\nu}\rangle = 0$ basis, we cannot have a neutral loop with one unit of lepton number violation on each of the propagators because if we turn the $\tilde{z}$ into a ν, we would also need $\partial_\mu\tilde{\nu} \to Z_\mu$. This is not possible, because the Goldstone boson is in the direction of the vevs, so has no $\tilde{\nu}$ component.

There are in addition two charged loops, drawn in figures 1a and 1d. Charged loops cannot have two gauge couplings at the incoming and outgoing vertices because $\Delta L = 2$ on a charged line is forbidden by charge conservation. It is not possible to have $\Delta L = 1$ on each line for the same reason as for the neutral gauge loop. One gauge vertex will produce a charged slepton and a wino/chargino; if $\tilde{w}^+ \to \ell$ by a $\Delta L = 1$ mass, then $\partial^\mu \tilde{L}^+ \to W^\mu$ would be required on the bosonic line. The

Goldstone boson has no $\tilde{L}^+$ component in the $\langle\tilde{\nu}\rangle = 0$ basis. For charged loops we can have two Yukawa/trilinear couplings at the vertices, which is the canonical trilinear diagram of figures 1a.

It is important to note that charged loops can have one gauge and one Yukawa coupling at the vertices. We show this contribution in figure 1d. To have one gauge and one Yukawa/trilinear coupling, we need (charged) gaugino-lepton mixing on the fermion line. This means we need $\delta_\mu \neq 0$, so these diagrams only contribute when we make assumption C. We will discuss the supersymmetric partner of diagram 1d, which could arise if W^μ mixes with $\partial^\mu \tilde{E}^c$, in a subsequent publication.

In the following subsections we make basis-independent estimates of the size of each loop, assuming that the Higgs vacuum expectation value, Higgs masses, slepton masses and chargino/neutralino masses are all of the same order $\sim m_{\rm SUSY}$. We will calculate the dependence of the neutral loop on the masses of the propagating particles in section 4. We also present estimates in the basis where the sneutrinos do not have a vev, $\langle\tilde{\nu}\rangle = 0$. A more detailed discussion of the diagrams involving charged loops will be given in a subsequent publication.

3.1 $\delta_\mu, \delta_B \sim 0$: the canonical loop

Figures 1a and 1b contain the usual diagrams considered in the literature contributing to one loop neutrino masses when we allow the lepton number violation to occur only at point II and VII. We neglect the $R\!\!\!/_p$ bilinear masses in this section, so the only available sources of $R\!\!\!/_p$ are the trilinear couplings at the vertices. Lines 1,2 of table 1 list the basis-independent coupling constant combinations to which these diagrams are proportional and estimate the contribution to the neutrino mass matrix. Note that on both the scalar and the fermion lines the particle must flip between doublet and singlet via an interaction with the Higgs/slepton vev. We represent this as a mass insertion of the vev to assist in writing basis-independent estimates.

To get the estimate in column 4 of table 1, we assume that we have some incident ν_i, in some choice of basis. At vertex II, we have a trilinear coupling λ^{iJk}, where in a generic basis we allow $J : 1 \ldots 4$ and $k : 1 \ldots 3$. A slepton E^c_k propagates to the top of the loop, where a trilinear mass insertion $A^{RTk} v_R + \lambda^{RTk} \mu_R v_u$ turns the E^c_k into an L_T. We again generically allow $T : 4 \ldots 1$ — this means we have not chosen the singlet charged lepton basis. L_T propagates to the vertex VII. Now following the fermion line down, we propagate an ℓ_J to the bottom of the loop, flip it into an SU(2) singlet via the mass $v_M \lambda^{JMn}$, and propagate e^c_n to vertex VII, where we put a trilinear λ^{Tjn}.

Thus, the contribution to the neutrino mass matrix is

$$m_{ij} \propto \frac{\lambda^{iJk}\lambda^{Tjn}}{16\pi^2 m^2_{\rm SUSY}} \lambda^{JMn} v_M \left[(\lambda A)^{RTk} v_R + v_u \mu_R \lambda^{RTk} \right], \tag{3.1}$$

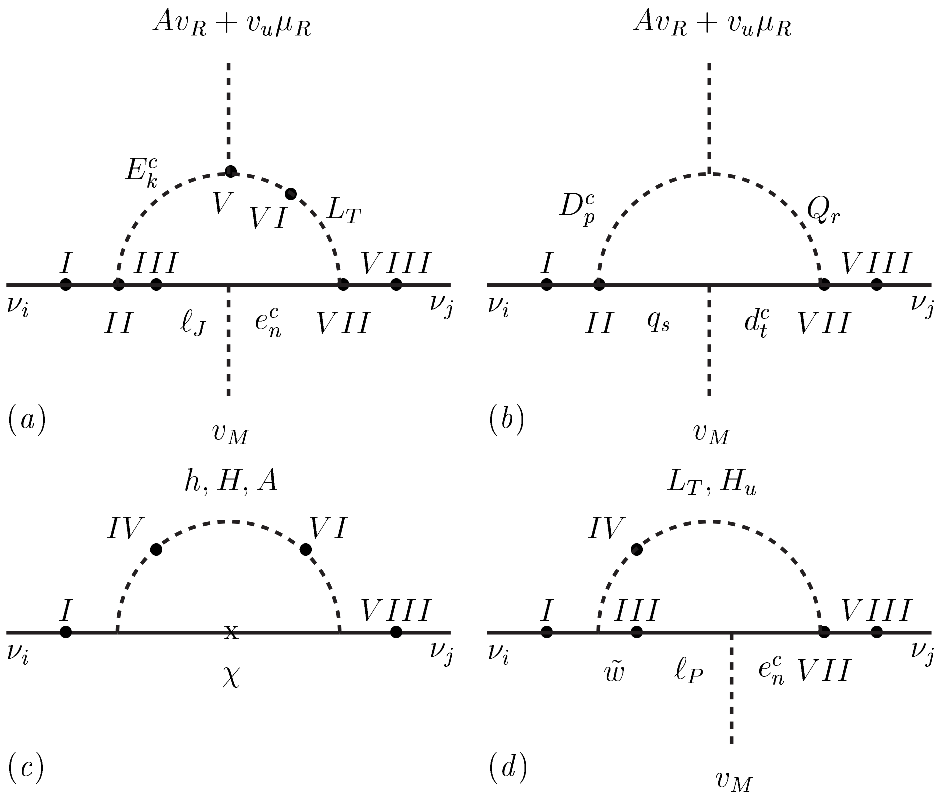

Figure 1: Schematic representation of one-loop diagrams contributing to neutrino masses, in a lagrangian basis. The blobs indicate possible positions for $\not{R}_p$ interactions, which can be trilinears (at positions II and VII) or mass insertions. The misalignment between $\vec{\mu}$ and $\vec{v}$ allows a mass insertion on the lepton/higgsino lines (at points I, III, or VIII) and at the A-term on the scalar line (position V). The soft $\not{R}_p$ masses appear as mass insertions at positions VI and IV on the scalar line. Figure a) is the charged loop with trilinear couplings λ (or h_e) at the vertices. Figure b) is the coloured loop with trilinear λ' or Yukawa h_b couplings. Figure c) is the neutral loop with two gauge couplings (this diagram is drawn in the MSSM mass eigenstate basis.), and figure d) is the charged loop with one gauge and and a Yukawa coupling. This diagram is relevant if gauginos mix with charged leptons — that is if $\delta_\mu \neq 0$.

where all repeated indices are summed. In basis where sneutrinos do not have a vev, and assuming that $(\lambda A)^{RTk} = A\lambda^{RTk}$[3] this is approximately

$$m_{ij} \sim \frac{\delta_\lambda^{ink}\delta_\lambda^{jkn}}{16\pi^2}\frac{m_n^e m_k^e}{m_{\rm SUSY}}\,, \tag{3.2}$$

where $m_n^e = |\vec{v}\cdot\lambda^n|$ are the charged lepton masses generated by the Higgs vev, for

[3] This avoids flavour violation among the sleptons. Additional diagrams would appear when one allows flavour violation in the slepton sector [15].

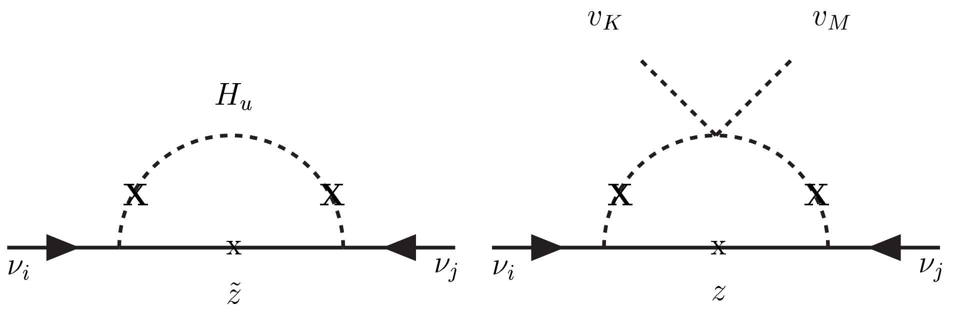

Figure 2: The Grossman-Haber contribution in interaction eigenstate basis. The scalars leaving the neutrino vertices are sneutrinos. The crosses on the scalar line are B_i, B_j, m^2_{4i} or m^2_{4j}.

$n : 1 \ldots 3$. We can compare this expression to the more correct formula given by

$$m_{ij} \sim \left\{\lambda^{ink}\lambda^{jkn} m^e_n m^e_k (A + \mu \tan\beta)\right\} \frac{f(L_k)}{16\pi^2 m^2_{E^c_k}}\,, \tag{3.3}$$

where $L_k = m^2_{L^k}/m^2_{E^c_k}$

$$f(x) = \frac{\ln x}{1-x}\,, \tag{3.4}$$

and $\mu = |\vec{\mu}|$.

Similarly, figure 1b with λ' couplings produces a neutrino mass matrix

$$m_{ij} \propto \frac{3\lambda'^{isp}\lambda'^{jrt}}{16\pi^2 m^2_{\rm SUSY}} \lambda'^{Mst} v_M ((\lambda A)'^{Rrp} v_R + v_u \mu_R \lambda'^{Rrp})\,. \tag{3.5}$$

Recall that we chose the quark basis to diagonalise the mass matrix $[\vec{v}\cdot\lambda']^{pq}$. The exact result is

$$m_{ij} \sim 3\lambda'^{ips}\lambda'^{jsp} m^d_p m^d_s (A + \mu\tan\beta) \frac{f(Q_p)}{16\pi^2 m^2_{D^c_p}}\,, \tag{3.6}$$

where $Q_p = m^2_{Q^p}/m^2_{D^c_p}$.

3.2 $\delta_\mu \sim 0$, $\delta_B \neq 0$: the GH loop

As mentioned above, there can be a loop contribution of order g^2 to the mass of the neutrinos that are massless at tree level from the diagram of figure 1c. This has been discussed in detail be Grossman and Haber [7] (see also [16, 12]). The soft SUSY breaking terms B_k and m^2_{4k}, and the sneutrino vev, mix the Higgses with the sleptons. If the soft masses are universal and $B_k = B\mu_k$, diagram 1c will be a loop correction to the mass of the neutrino that is massive at tree level. In figure 3 we show the supersymmetric partner of this diagram which gives a small contribution to the mass of the neutrino which is massive at tree level. However if $\vec{B}$ is misaligned with respect to both $\vec{v}$ and $\vec{\mu}$, the diagram of figure 1c will give a mass also to the neutrinos that are massless at tree level.

The diagram in figure 1c is in the "MSSM mass eigenstate basis",where the sneutrino does not have a vev. The sneutrinos mix with the neutral CP-even Higgses h, H and the CP-odd Higgs A through B_k and m^2_{4k}. In section 4, we calculate this diagram in the mass insertion approximation, and show that we get the same answer as by expanding the exact result of [12]. We also show the contributions to this diagram in an arbitrary basis in figure 2, because this facilitates basis-independent estimates. There is an additional diagram (not drawn) combining those of figure 2, with one unit of lepton number violation from B and one from $\tilde{m}^2$. In an arbitrary interaction eigenstate basis, the sneutrino could have a vev, which induces lepton number violating masses through the D-terms. This would correspond to figure 2 with diagonal $\tilde{m}^2$ and L violation from the sneutrino vevs.

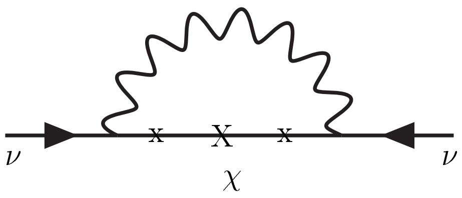

Figure 3: Feynman diagram showing the gauge loop correction to the tree-level neutrino mass in MSSM neutralino mass eigenstate basis. This is the supersymmetric partner of the GH loop. The small "x"s on the internal fermion line correspond to the interaction $\mu_i \nu_i h_u$. The large X is a neutralino mass. There would be additional diagrams in a basis where $\langle \tilde{\nu} \rangle \neq 0$.

Figure 2 and the diagrams with one unit of lepton number violation from B and one from $\tilde{m}^2$ contribute to the neutrino mass matrix by

$$m_{ij} \propto g^2 \left(v_u B_i + m^2_{iK} v^K \right) \left(v_u B_j + v^M m^2_{Mj} \right) . \tag{3.7}$$

We can write $v_J [m_L]^2_{Ji}$ in terms of B_i using the minimisation conditions for the Higgs potential [12]:

$$v_u B_J + (\delta_{JK} m_Z^2 \cos 2\beta + [m_L]^2_{JK}) v^K = 0 \, , \tag{3.8}$$

where at tree level $[m_L]^2_{KJ} = \tilde{m}^2_{JK} + \mu_J \mu_K$, and B and $\tilde{m}^2$ are defined in equation (1.2).

In section 4, we calculate diagram 1c, and find that (in the lepton flavour basis where the sneutrino has no vev) it is approximately

$$m_{ij} \sim \frac{g^2 \delta^i_B \delta^j_B}{64\pi^2} m_\chi \, . \tag{3.9}$$

Note that we neglect the supersymmetric partner of these diagrams, figure 3, because it is a small correction to the tree-level mass. As explained above, the GH diagram gives a mass to a different combination of neutrinos if B_J is not exactly aligned with μ_J, that is $B_J \neq B\mu_J$. So in the basis where the sneutrino has no vev, we are only interested in the component of B_i which is orthogonal to μ_i. Writing

$$B^i - \frac{B^i \mu_i}{\sum_i \mu_i^2} \mu^i \equiv B^i_\perp = B^i - B^i_\parallel \, , \tag{3.10}$$

we find

$$m_{ij} \sim \frac{g^2(\delta^i_{B_\perp}\delta^j_{B_\parallel} + \delta^i_{B_\parallel}\delta^j_{B_\perp} + \delta^i_{B_\perp}\delta^j_{B_\perp})}{64\pi^2} m_\chi \,. \tag{3.11}$$

The component of B_i parallel to μ_i contributes to the mass of the neutrino that is massive at tree level. We also neglect the diagram $\sim \delta^i_\mu \delta^j_\mu h_t^4$. This comes from mixing the neutrino with the up-type higgsino, $\tilde{h}_u$, which has a Majorana mass from a loop like figure 1b with tops and stops instead of bottoms/sbottoms.

There is an additional contribution to the neutrino mass matrix in the case that $h\delta_B \sim \delta_\lambda$. This comes from the diagram of figure 1a, with an L violating mass insertion at point VI. One unit of lepton number violation comes from a trilinear, and another from B_k or m^2_{4k}. We use the minimisation condition for the Higgs/slepton potential: $-\tan\beta B_k = \tilde{m}^2_{4k}$ (in the $\langle\tilde{\nu}\rangle = 0$ basis) [12], to get the neutrino mass estimate in table 1.

3.3 Contributions when $\delta_\mu \neq 0$, $\delta_B \neq 0$

In this section, we include L violating mass insertions on all possible propagators for diagrams with charged particles in the loops. Consider first the diagram of figure 1a. On the scalar propagator there can be a mass insertion δ_B discussed at the end of the previous section, or δ_μ (the latter appears in the off-diagonal mass term mixing the E^c with L or H). Both external neutrino legs and the internal lepton line can have mass insertions δ_μ. Each of these possibilities is represented by a blob on the diagrams of figure 1. The diagram is shorthand for a series of loops. The possibilities are listed in table 1, along with the corresponding estimates for m_ν.

The diagrams must have two units of same-sign lepton number violation to generate a neutrino mass. Diagrams with $\Delta L = +1$ and $\Delta L = -1$ would contribute to wavefunction renormalisation. In diagram 1a, this implies that we need one unit of lepton number violation from I–III and one from IV –VIII. Schematically we expect terms of order

$$(\delta_\mu + \delta_\lambda + \delta_\mu)(\delta_\mu + \delta_B + \delta_\lambda + \delta_\mu)\,, \tag{3.12}$$

where the first parenthese corresponds to the blobs I–III and the second to the blobs IV–VIII. We neglect for convenience in the table the m^2_{4k} insertions because these can be rewritten in terms of B_i using the minimisation conditions [12]. The contributions of order δ^2_λ, $\delta^2_{\lambda'}$, $\delta_\lambda\delta_B$ have already been discussed, the corresponding estimates are given on the first, second and fifth lines of table 1, respectively. We briefly discuss here how we calculated the results presented in table 1 for the new diagrams which we consider.

We first include the δ_μ perturbations on the external lines at points I and VIII. These will be present for both λ (figure 1a) and λ' (figure 1b) and correspond to expressions on lines 4 and 6 of table 1. We neglect the diagram with mass insertions on both legs because this will be proportional to the tree level mass $\sim \mu_i\mu_j/m_\chi$. We can estimate the diagram with lepton number violation at points I and VII as follows. In the $\langle\tilde{\nu}_k\rangle = v_k = 0$ basis ($k : 1 \ldots 3$), the incident neutrino ν_i will be in the direction $(\hat{\nu}_i)^J = v_M \lambda^{MJi}/m^e_i$. The mass insertion on the external leg is $\mu_i = \vec{v} \cdot \lambda^i \cdot \vec{\mu}/m^e_i \propto \delta^i_\mu$, and allows the incident neutrino to turn into a Higgsino h_u. In an arbitrary basis, where $v_i \neq 0$ is allowed, the mass insertion will be the $\mu_i v_4 - v_i \mu_4 \propto \vec{\mu} \cdot \lambda^i \cdot \vec{v}$. Continuing along the incident line, in the basis where the sneutrinos do not have vevs, the h_u can turn into an h_d via μ_4, and at the vertex h_d interacts with E^c and ℓ via a Yukawa coupling. In an arbitrary basis, the neutralino after the mass insertion of μ_i or v_i will be an h_u or a gaugino. These can respectively be turned into the linear combination of ℓ_I corresponding to the h_d by μ_I or v_I. These vectors are misaligned by an amount δ_μ, which we can neglect here because we have one unit of L violation from the mass insertion. We assume $|\vec{v}| \sim |\vec{\mu}| \sim m_{\mathrm{SUSY}}$. In column four of table 1, we therefore write the vertex as $\mu_I \lambda'^{Ipq}[\mu_I \lambda^{IJk}]$ for the squark [charged] loop. The remainder of the diagram is the same as lines 1 and 2 of the table.

Additional terms of order $\delta_\mu(\delta_\mu + \delta_\lambda)$ are obtained when we include the misalignment between $\vec{\mu}$ and $\vec{v}$ at point V of diagram 1a, giving the contributions on lines 7-9 of table 1. We can express

$$\vec{\mu} = \frac{\vec{\mu} \cdot \vec{v}}{v^2} \vec{v} + \frac{\vec{\mu} \cdot \lambda^\ell \cdot \vec{v}}{m^2_\ell} \vec{v} \cdot \lambda^\ell \, , \tag{3.13}$$

or alternatively the part of $(v_R A + \mu_R v_u)\lambda^{RTk}$ that is misaligned with respect to $v_R \lambda^{RTk}$ can be written as

$$v_u \frac{\mu_R \lambda^{RP\ell} v_P v_Q \lambda^{QS\ell} \lambda^{STk}}{m^2_\ell} \, . \tag{3.14}$$

Finally there are corrections proportional to δ_μ from μ_i mass insertions on the internal lines of figure 1a. We assume $\vec{v}_J \lambda^{JLi} \lambda^{LMk} \vec{v}_M = \delta^{ik} m_i m_k$ (because $v - \mu$ misalignment is higher order effect) to get the results listed in lines 9–13 of table 1.

If $\delta_\mu \neq 0$, the $\tilde{w}$ mixes with the charged leptons, so there are loop diagrams with a gauge coupling at one end and a Yukawa coupling at the other, e.g. see figure 1d. These gauge $\times$ Yukawa diagrams can only appear if the gaugino mixes with the lepton on the internal line, so they are proportional to δ_μ. The other unit of L violation must be due to δ_μ (at points I or VIII) or δ_B (at point IV). These contributions are estimated in lines 14-16 of table 1. There is no contribution with a trilinear λ at the vertex VII because it gives $[m^{ij}_\nu] \sim \lambda^{ijk}$, and λ^{ijk} is anti-symmetric on indices ij whereas the mass matrix is symmetric.

$No.$	diagram	position of $\not{R}_p$	$16\pi^2 m_{SUSY}^2 \times m_\nu^{ij}$	$16\pi^2 m_{SUSY}[m_v]^{ij} (<\tilde{\nu}>= 0$ basis)
1	a	$II \quad VII$	$\lambda^{iJk}\lambda^{jTn}\lambda^{JMn}v_M[(\lambda A)^{RTk}v_R + v_u\mu_R\lambda^{RTk}]$	$\delta_\lambda^{ink}\delta_\lambda^{jkn}m_n^e m_k^e$
2	b	$II \quad VII$	$3\lambda'^{isp}\lambda'^{jrt}\lambda'^{Mst}v_M((\lambda A)'^{Rrp}v_R + v_u\mu_R\lambda'^{Rrp})$	$3\delta_{\lambda'}^{iqq}\delta_{\lambda'}^{jqq}(m_q^d)^2$
3	c	$IV \quad VI$	$g^2(v_uB^i + m_{iK}^2 v^K)(v_uB^j + v^M m_{Mj}^2)/(4m_{SUSY}^3)$	$g^2\delta_B^i\delta_B^j m_\chi m_{SUSY}/4$
4	b	$I \quad VII + II \quad VIII$	$3(v_A\lambda^{ABi}\mu_B)\lambda'^{jrt}\mu_I\lambda'^{Isp}\lambda'^{Mst}v_M((\lambda A)'^{Rrp}v_R + v_u\mu_R\lambda'^{Rrp})/m_{SUSY}$	$3(\delta_\mu^i\delta_{\lambda'}^{jqq} + \delta_\mu^j\delta_{\lambda'}^{iqq})(m_q^d)^2 h_q^d$
5	a	$II \quad VI$	$\lambda^{iJk}\lambda^{Tjn}\lambda^{JMn}v_M(v_RA + \mu_Rv_u)\lambda^{RNk}B_NB_T/m_{SUSY}^2$	$\delta_\lambda^{ijk}m_k^e\delta_B^k(m_j^eh_j^e - m_i^eh_i^e)$
6	a	$I \quad VII + II \quad VIII$	$(v_A\lambda^{ABi}\mu_B)\lambda^{Tjn}\mu_I\lambda^{IJk}\lambda^{JMn}v_M[(\lambda A)^{RTk}v_R + v_u\mu_R\lambda^{RTk}]/m_{SUSY}$	$(\delta_\mu^i\delta_\lambda^{jkk} + \delta_\mu^j\delta_\lambda^{ikk})(m_k^e)^2h_k^e$
7	a	$I \quad V$	$(v_A\lambda^{ABi}\mu_B)\lambda^{Tjn}\mu_I\lambda^{IJk}\lambda^{JMn}v_M\frac{\delta_\mu^\ell v_Q\lambda^{QS\ell}\lambda^{STk}}{m_\ell m_{SUSY}}$	$\delta_\mu^i\delta_\mu^j((m_j^e)^2h_j^e + (m_i^e)^2h_i^e)$
8	a	$II \quad V$	$v_u\mu\lambda^{iJk}\lambda^{JMn}v_M\lambda^{Tjn}\frac{\delta_\mu^\ell v_Q\lambda^{QS\ell}\lambda^{STk}}{m_\ell}$	$\delta_\lambda^{ijk}\delta_\mu^k m_k^e(h_i^em_i^e - h_j^em_j^e)$
9	a	$III \quad V$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M]\lambda^{Tjn}\frac{\delta_\mu^\ell v_Q\lambda^{QS\ell}\lambda^{STk}}{m_\ell}$	$\delta_\mu^i\delta_\mu^j m_j^em_i^eh_i^eh_j^e$
10	a	$III \quad VIII$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M](v_RA + \mu_Rv_u)\lambda^{RTk}\lambda^{TJn}\mu_J(v_A\lambda^{ABj}\mu_B)/m_{SUSY}^5$	$\delta_\mu^i\delta_\mu^j((m_i^eh_i^e)^2 + (m_j^eh_j^e)^2)$
11	a	$I \quad VI$	$(v_A\lambda^{ABi}\mu_B)\mu_I\lambda^{IJk}\lambda^{JMn}v_M(v_RA + \mu_Rv_u)\lambda^{RNk}B_NB_T\lambda^{Tjn}/m_{SUSY}^7$	$\delta_\mu^i\delta_B^j(m_j^eh_j^e)^2 + \delta_\mu^j\delta_B^i(m_i^eh_i^e)^2$
12	a	$III \quad VII$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M](v_RA + \mu_Rv_u)\lambda^{RTk}\lambda^{Tjn}/m_{SUSY}^2$	$\delta_\lambda^{jin}\delta_\mu^n m_n^e(m_j^eh_j^e - m_i^eh_i^e)$
13	a	$III \quad VI$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M](v_RA + \mu_Rv_u)\lambda^{RNk}B_NB_T\lambda^{Tjn}/m_{SUSY}^6$	$g(\delta_B^i\delta_\mu^jh_j^eh_i^em_i^em_j^e + \delta_B^j\delta_\mu^ih_i^eh_j^em_j^em_i^e)$
14	d	$III \quad IV$	$gB_i[\mu_P\lambda^{PMn}v_M]B_T\lambda^{Tjn}/m_{SUSY}^3$	$g(\delta_B^i\delta_\mu^j(m_j^e)^2 + \delta_B^j\delta_\mu^i(m_i^e)^2)$
15	d	$III \quad VIII$	$g[\mu_P\lambda^{PMn}v_M]\lambda^{iJn}\mu_J(v_A\lambda^{ABj}\mu_B)/m_{SUSY}^2$	$g\delta_\mu^i\delta_\mu^j((m_i^e)^2 + (m_j^e)^2)$
16	d	$I \quad III$	$g(v_A\lambda^{ABi}\mu_B)[\mu_P\lambda^{PMn}v_M]\lambda^{jJn}\mu_J/m_{SUSY}^2$	$g\delta_\mu^i\delta_\mu^j((m_i^e)^2 + (m_j^e)^2)$

Table 1: Estimated contributions to $[m_\nu]^{ij}$ from all the diagrams. In the second two columns is the label of the diagram of figure 1, and the position on the diagram of the two $\Delta L = 1$ interactions. Column four is the "basis independent" combination of coupling constants for the diagram (which must be symmetrised on $i \leftrightarrow j$), and column five is the estimated contribution to the neutrino mass matrix (in the "flavour basis" where $\langle\tilde{\nu}\rangle = 0$ and $\hat{\nu}_i = \vec{v}\cdot\lambda^i/m_i^e$). All indices other than i and j are summed.

4. The GH loop in the MSSM mass eigenstate basis

In this section, we calculate the one loop neutrino mass induced by slepton-higgs mixing (figure 1c). We first do this using mass insertions the MSSM mass eigenstate basis, then compare to our previous exact (and basis independent) result in one generation [12]. The two calculations agree, but differ slightly from the original calculation of [7].

In the $\langle\tilde{\nu}\rangle = 0$ basis the two units of lepton number violation come from B_i and $[m_L]^2_{4i}$ (the crosses on the scalar propagators in figure 1c), which mix the sneutrino with the Higgses. The scalars that leave the vertex where the neutrinos go in are sneutrinos. We can write $[m_L]^2_{4i} = -\tan\beta B_i$, using equation (3.8) in the basis where $\langle\tilde{\nu}\rangle = 0$.

The amplitude for this diagram connecting a neutrino ν_i to ν_j is

$$\begin{aligned}
\mathcal{A}_{ij} = B_i B_j \frac{g^2}{4} \sum_{\chi\alpha} m_{\chi_\alpha} \left(Z_{\alpha 2} - \frac{Z_{\alpha 1} g'}{g} \right)^2 \times \\
\times \big\{ (\cos^2\alpha + 2\tan\beta\cos\alpha\sin\alpha + \tan^2\beta\sin^2\alpha) I(m_h^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) + \\
+ (\sin^2\alpha - 2\tan\beta\cos\alpha\sin\alpha + \tan^2\beta\cos^2\alpha) I(m_H^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) - \\
- (\cos^2\beta + 2\tan\beta\cos\beta\sin\beta + \tan^2\beta\sin^2\beta) I(m_A^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) \big\} . \quad (4.1)
\end{aligned}$$

Recall $\{\chi_\alpha\}$ for $\alpha : 7 \ldots 4$ are the MSSM neutralinos, and h, H, and A are the usual MSSM fields:

$$\begin{aligned}
h &= \cos\alpha H_u^R - \sin\alpha H_d^R \\
H &= \sin\alpha H_u^R + \cos\alpha H_d^R \\
A &= \cos\beta H_u^I - \sin\beta H_d^I ,
\end{aligned} \quad (4.2)$$

(H_u^R and H_u^I are the real and imaginary parts of the neutral components of the MSSM Higgs fields) and

$$\begin{aligned}
I_{1gen}(m_s^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m_{\tilde{\nu}}^2} \frac{1}{k^2 + m_s^2} \frac{1}{k^2 + m_{\tilde{\nu}}^2} \frac{1}{k^2 + m_{\chi_\alpha}^2} \\
&= \frac{1}{16\pi^2} \frac{1}{m_{\tilde{\nu}}^2 - m_s^2} \left\{ \frac{1}{m_{\chi_\alpha}^2 - m_{\tilde{\nu}}^2} - \frac{m_{\chi_\alpha}^2}{(m_{\chi_\alpha}^2 - m_{\tilde{\nu}}^2)^2} \ln \frac{m_{\chi_\alpha}^2}{m_{\tilde{\nu}}^2} + \right. \\
&\quad + \frac{m_{\tilde{\nu}}^2}{(m_{\chi_\alpha}^2 - m_{\tilde{\nu}}^2)(m_{\tilde{\nu}}^2 - m_s^2)} \ln \frac{m_{\chi_\alpha}^2}{m_{\tilde{\nu}}^2} - \\
&\quad \left. - \frac{m_s^2}{(m_{\chi_\alpha}^2 - m_s^2)(m_{\tilde{\nu}}^2 - m_s^2)} \ln \frac{m_{\chi_\alpha}^2}{m_s^2} \right\} . \quad (4.3)
\end{aligned}$$

This gives

$$\begin{aligned}
m_\nu^{ij} = \frac{g^2 B^i B^j}{4\cos^2\beta} \sum_{\chi\alpha} \left(Z_{\alpha 2} - \frac{Z_{\alpha 1} g'}{g} \right)^2 m_{\chi_\alpha} \big\{ I(m_h^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) \cos^2(\alpha - \beta) + \\
+ I(m_H^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) \sin^2(\alpha - \beta) - I(m_A^2, m_{\tilde{\nu}}^2, m_{\chi_\alpha}^2) \big\} , \quad (4.4)
\end{aligned}$$

which does not quite match Grossman and Haber's result, but goes to the same limits when various masses become large. We can check equation (4.4) by expanding the exact result of [12]. The result of [12] is for one lepton generation only; we assume here that the sneutrinos are degenerate, so we can compare to this calculation. From [12] we have:

$$m_\nu = \frac{g^2}{64\pi^2}\sum_{\alpha:7\ldots4} m_{\chi\alpha}\left(Z_{\alpha2} - \frac{Z_{\alpha1}g'}{g}\right)^2 \sum_n (\hat\nu\cdot\hat s_n)^2 \epsilon_n B_0(0, M_n^2, m_{\chi_\alpha}^2)\,, \tag{4.5}$$

where ϵ_n is $+1$ for s_n a CP-even scalar, and -1 for n a CP-odd. Note that in an $\not\!R_p$ theory, the sneutrinos split into a CP-even and a CP-odd scalar, that are not mass degenerate. $\hat\nu$ is the lepton direction corresponding to the incident neutrino, and $\hat s_n$ is the direction in Higgs-slepton space corresponding to the mass eigenstate s_n. B_0 is a Passarino-Veltman function:

$$\begin{aligned} B_0(0, M_s^2, m_\chi^2) &= -16\pi^2 i \lim_{q\to0}\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}}\frac{1}{[(k+q)^2 - m_\chi^2](k^2 - M_s^2)} \\ &\supset -\frac{M_s^2}{M_s^2 - m_\chi^2}\ln\left(\frac{M_s^2}{m_\chi^2}\right) = I(M_s^2, m_\chi^2)\,. \end{aligned} \tag{4.6}$$

There are divergent and scale-dependent contributions to B_0 in addition to $I(M_s^2, m_\chi^2)$; however these cancel in the sum over scalars s_i in equation (4.5).

To compare this result to equation (4.4), we need to expand the mixing angles $(\hat\nu\cdot\hat s_n)$ and the masses M_n^2 in B_i. We define the neutrino direction to be $\hat\nu_i \equiv \vec v\cdot\lambda^i/m_i$ (this is the defintion of the charged lepton direction we have been using all through the paper).

As $\not\!R_p \to 0$, we make the following identifications:

$$\begin{array}{ll} s_5 = h_5 \to h & \\ s_4 = h_4 \to H & s_9 = A_4 \to A \\ s_3 = h_3 \to \tilde\nu_\tau^R & s_8 = A_3 \to \tilde\nu_\tau^I \\ s_2 = h_2 \to \tilde\nu_\mu^R & s_7 = A_2 \to \tilde\nu_\mu^I \\ s_1 = h_1 \to \tilde\nu_e^R & s_6 = A_1 \to \tilde\nu_e^I, \end{array}$$

where h_n is CP-even, A_n is CP-odd, $\tilde\nu^R$ is the real part of the sneutrino, and the right hand side of the arrow are MSSM fields.

The diagram that generates a Majorana mass for a neutrino must contain two units of lepton number violation. There is a contribution from the mixing angles between the neutrinos and the $s_n, n: 9\ldots1$, and from the h_k — A_k mass differences for $k: 3\ldots1$. We consider first the contributions from h_5, h_4 and A_4. The mixing angles $(\hat\nu_i\cdot\hat s_n)$ for $s_n = h_5, h_4$ or A_4 will be proportional to B_i, so $\not\!R_p$ corrections to the h_5, h_4 or A_4 masses can be neglected in expanding equation (4.5). They would be a higher order effect. We find, for instance,

$$\hat\nu_i\cdot\hat h_5 = \frac{B_i(\cos\alpha + \sin\alpha\tan\beta)}{m_h^2 - m_{\tilde\nu_i}^2}\,, \tag{4.7}$$

which substituted into (4.5) gives the contribution of the last line of equation (4.3) to the complete result.

The contribution from h_3 and A_3 (scalars that are mostly $\tilde{\nu}_\tau$) can be written

$$\frac{g^2}{64\pi^2}\sum_{\alpha:7...4} m_{\chi_\alpha}\left(Z_{\alpha 2}-\frac{Z_{\alpha 1}g'}{g}\right)^2\Big\{I(M^2_{h_3},m^2_{\chi_\alpha})-I(M^2_{A_3},m^2_{\chi_\alpha})- \tag{4.8}$$
$$-\left[(\hat{\nu}_\tau\cdot\hat{h}_5)^2+(\hat{\nu}_\tau\cdot\hat{h}_4)^2-(\hat{\nu}_\tau\cdot\hat{A}_4)^2\right]I(M^2_{\tilde{\nu}_\tau},m^2_{\chi_\alpha})\Big\}.$$

The first line is the contribution from the $\tilde{\nu}^R-\tilde{\nu}^I$ mass difference, and the second line is from the mixing angle: $(\hat{\nu}_\tau\cdot\hat{h}_3)^2=1-(\hat{\nu}_\tau\cdot\hat{h}_5)^2-(\hat{\nu}_\tau\cdot\hat{h}_4)^2$. The third term of equation (4.8) gives contribution from the second last term of equation (4.3) to the complete result, and the first two terms of equation (4.8) give the contribution from the first two terms of equation (4.3). This can be seen by writing $m^2_{h_3}=m^2_{\tilde{\nu}_\tau}+\Delta m^2_{\tilde{\nu}_\tau}/2$, and

$$\Delta m^2_{\tilde{\nu}_\tau}=\left(-\frac{B_3^2(\cos\alpha+\sin\alpha\tan\beta)^2}{m_h^2-m_{\tilde{\nu}}^2}-\frac{B_3^2(\sin\alpha-\cos\alpha\tan\beta)^2}{m_H^2-m_{\tilde{\nu}}^2}+\right.$$
$$\left.+\frac{B_3^2(\cos\beta-\sin\beta\tan\beta)^2}{m_A^2-m_{\tilde{\nu}}^2}\right). \tag{4.9}$$

The result in [7] corresponds to this contribution.

We can make a similar expansion for $\hat{\nu}_\mu\cdot h_2$ and $\hat{\nu}_\mu\cdot A_2$, and for $\hat{\nu}_e$ assuming as here that there is no flavour violation amoung the sneutrinos, e.g. $(\hat{\nu}_\tau\cdot\hat{A}_2)=0$, etc.

5. Discussion

We now return to our introductory suggestion that neglecting $R\!\!\!/_p$ masses in the loops creates confusion about basis. Consider the superpotential for a one-generation model. This could have two $R\!\!\!/_p$ parameters: λ' or μ_3. As discussed after equation (1.3), one can define the Higgs such that $\mu_3=0$ or such that $\lambda'=0$. In the basis where $\mu_3=0$, there is the usual loop neutrino mass corresponding to diagram 1b with $R\!\!\!/_p$ at the vertices II and VII. Now consider the basis where $\lambda'=0$. If the mass μ_3 is neglected in the loops, then it appears that there is no loop neutrino mass. This is perplexing, because we know that the tree-level mass is proportional to the misalignment between the Higgs-slepton vev and (μ_4,μ_3). If we are in a model where $\vec{\mu}\parallel\vec{v}$, the basis rotation cannot have moved the neutrino mass from loop to tree level. Thus, it is unclear where did the contribution to the mass go. If we include the loop 1b with $R\!\!\!/_p$ at points I and VIII, this confusion is removed.

We conclude that the basis in which neutrino mases are calculated is *irrelevant*, providing that all the contributions are included. We believe that "basis-related" confusion comes from neglecting the $R\!\!\!/_p$ masses in computing loop neutrino masses: the magnitude of the contribution being neglected depends on the basis choice.

We therefore sit in the basis where the sneutrino has no vev, and consider the size of different contributions and the structure of the neutrino mass matrix in lepton flavour space. The tree-level mass contribution to the neutrino mass matrix is

$$[m_\nu]_{ij} \sim \frac{\mu_i \mu_j}{m_\chi}\,, \tag{5.1}$$

in all cases.

In case A, where we neglect $R\!\!\!/_p$ masses in the loops, the loop contributions from the squark/quark and lepton/slepton loops are respectively of order

$$[m_\nu]^{ij} \sim 3\frac{\lambda'^{iqq}\lambda'^{jqq}(m_q^d)^2}{16\pi^2 m_{\rm SUSY}} + \frac{\lambda^{ik\ell}\lambda^{j\ell k} m_\ell^e m_k^e}{16\pi^2 m_{\rm SUSY}}\,. \tag{5.2}$$

Note that the quark basis can be chosen to simultaneously diagonalise the λ' and down-type Yukawa couplings, but that the lepton basis that diagonalises the lepton mass matrix $\lambda^{IJk}v_I$ does not neccessarily diagonalise $\lambda^{i\ell k}$ on indices ℓk.

In case B, we include the soft masses $\{B_i\}$ and $\{m_{4i}^2\}$ in the GH diagram (figure 1c) which contributes

$$[m_\nu]^{ij} \sim \frac{g^2}{64\pi^2 m_{\rm SUSY}}\left(B_i^\perp B_j^{||} + B_i^{||}B_j^\perp + B_i^\perp B_j^\perp\right) m_\chi\,, \tag{5.3}$$

where $B_i^\perp$ and $B_j^{||}$ are defined in equation (3.10), and we have neglected the term of order $B_i^{||}B_j^{||}$ because it is aligned with the tree-level mass of equation (5.1).

In case C, we include all the $R\!\!\!/_p$ masses in perturbation theory. All the terms listed in column 5 of table 1 will contribute. Those from diagram 1d, which involves a gauge coupling, are potentially largest:

$$[m_\nu]^{ij} \sim \frac{g}{16\pi^2 m_{\rm SUSY}}[(m_e^i)^2 + (m_e^j)^2]\frac{\mu^i\mu^j}{|\mu|^2}\,. \tag{5.4}$$

Note that this is *not* aligned with the tree-level mass due to the inclusion of lepton masses. There are additional contributions of order $h^4\mu_i\mu_j/|\mu|^2$ or $h^4 B_i\mu_j/(|\mu|B)$ from mass insertions on diagrams 1a and 1b. (h is some Yukawa coupling.) These are listed in table 1.

To summarize we have estimated one-loop contributions from R-parity violating parameters to the neutrino mass matrix in the context of the MSSM. We included the $R\!\!\!/_p$ bilinears in the mass insertion approximation, which introduces new loops and new contributions to the canonical graphs considered in the literature. For the neutral graph arising from slepton-Higgs mixing, our mass insertion results are in agreement with the perturbative expansion of the exact result.

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Erratum

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The mass dimensions in column four of our original table were eratically wrong. We correct them here, and also two typographical errors in column 5 on lines 7 and 13.

No.	diagram	position of $R\!\!\!/_p$	$16\pi^2 m^2_{\mathrm{SUSY}} \times m^{ij}_\nu$	$16\pi^2 m_{\mathrm{SUSY}}[m_v]^{ij}$ ($\langle\tilde\nu\rangle = 0$ basis)
1	a	$II\quad VII$	$\lambda^{iJk}\lambda^{jTn}\lambda^{JMn}v_M[(\lambda A)^{RTk}v_R+v_u\mu_R\lambda^{RTk}]$	$\delta^{ink}_\lambda\delta^{jkn}_\lambda m^e_n m^e_k$
2	b	$II\quad VII$	$3\lambda'^{isp}\lambda'^{jrt}\lambda'^{Mst}v_M[(\lambda A)'^{Rrp}v_R+v_u\mu_R\lambda'^{Rrp}]$	$3\delta^{iqq}_{\lambda'}\delta^{jqq}_{\lambda'}(m^d_q)^2$
3	c	$IV\quad VI$	$g^2(v_uB^i+m^2_{iK}v^K)(v_uB^j+v^Mm^2_{Mj})/(4m^3_{\mathrm{SUSY}})$	$g^2\delta^i_B\delta^j_B m_\chi m_{\mathrm{SUSY}}/4$
4	b	$I\ \ VII+II\ \ VIII$	$3(v_A\lambda^{ABi}\mu_B)\lambda'^{jrt}\mu_I\lambda'^{Isp}\lambda'^{Mst}v_M[(\lambda A)'^{Rrp}v_R+v_u\mu_R\lambda'^{Rrp}]/(m^2_{\mathrm{SUSY}}m^e_i)$	$3(\delta^i_\mu\delta^{jqq}_{\lambda'}+\delta^j_\mu\delta^{iqq}_{\lambda'})(m^d_q)^2h^d_q$
5	a	$II\quad VI$	$\lambda^{iJk}\lambda^{Tjn}\lambda^{JMn}v_M[v_R(A\lambda)^{RNk}+\mu_Rv_u\lambda^{RNk}]B_NB_T/m^4_{\mathrm{SUSY}}$	$\delta^{ijk}_\lambda m^e_k\delta^k_B(m^e_jh^e_j-m^e_ih^e_i)$
6	a	$I\ \ VII+II\ \ VIII$	$(v_A\lambda^{ABi}\mu_B)\lambda^{Tjn}\mu_I\lambda^{IJk}\lambda^{JMn}v_M[(\lambda A)^{RTk}v_R+v_u\mu_R\lambda^{RTk}]/(m^2_{\mathrm{SUSY}}m^e_i)$	$(\delta^i_\mu\delta^{jkk}_\lambda+\delta^j_\mu\delta^{ikk}_\lambda)(m^e_k)^2h^e_k$
7	a	$I\quad V$	$(v_A\lambda^{ABi}\mu_B)\lambda^{Tjn}\mu_I\lambda^{IJk}\lambda^{JMn}v_M\delta^\ell_\mu v_Q\lambda^{QS\ell}\lambda^{STk}/m_\ell m^e_i$	$\delta^i_\mu\delta^j_\mu((m^e_jh^e_j)^2+(m^e_ih^e_i)^2)$
8	a	$II\quad V$	$v_u\mu\lambda^{iJk}\lambda^{JMn}v_M\lambda^{Tjn}\delta^\ell_\mu v_Q\lambda^{QS\ell}\lambda^{STk}/m_\ell$	$\delta^{ijk}_\lambda\delta^k_\mu m^e_k(h^e_im^e_i-h^e_jm^e_j)$
9	a	$III\quad V$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M]\lambda^{Tjn}\delta^\ell_\mu v_Q\lambda^{QS\ell}\lambda^{STk}/m_\ell$	$\delta^i_\mu\delta^j_\mu m^e_jm^e_ih^e_ih^e_j$
10	a	$III\quad VIII$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M][v_R(A\lambda)^{RTk}+\mu_Rv_u\lambda^{RTk}]\lambda^{TJn}\mu_J(v_A\lambda^{ABj}\mu_B)/(m^4_{\mathrm{SUSY}}m^e_j)$	$\delta^i_\mu\delta^j_\mu((m^e_ih^e_i)^2+(m^e_jh^e_j)^2)$
11	a	$I\quad VI$	$(v_A\lambda^{ABi}\mu_B)\mu_I\lambda^{IJk}\lambda^{JMn}v_M[v_R(A\lambda)^{RNk}+\mu_Rv_u\lambda^{RNk}]B_NB_T\lambda^{Tjn}/(m^6_{\mathrm{SUSY}}m^e_i)$	$\delta^i_\mu\delta^j_B(m^e_jh^e_j)^2+\delta^j_\mu\delta^i_B(m^e_ih^e_i)^2$
12	a	$III\quad VII$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M][v_R(A\lambda)^{RTk}+\mu_Rv_u\lambda^{RTk}]\lambda^{Tjn}/m^2_{\mathrm{SUSY}}$	$\delta^{jin}_\lambda\delta^n_\mu m^e_n(m^e_jh^e_j-m^e_ih^e_i)$
13	a	$III\quad VI$	$\lambda^{iJk}\mu_J[\mu_P\lambda^{PMn}v_M][v_R(A\lambda)^{RNk}+\mu_Rv_u\lambda^{RNk}]B_NB_T\lambda^{Tjn}/m^6_{\mathrm{SUSY}}$	$(\delta^i_B\delta^j_\mu h^e_jh^e_im^e_im^e_j+$ $+\delta^j_B\delta^i_\mu h^e_ih^e_jm^e_jm^e_i)$
14	d	$III\quad IV$	$gB_i[\mu_P\lambda^{PMn}v_M]B_T\lambda^{Tjn}/m^3_{\mathrm{SUSY}}$	$g(\delta^i_B\delta^j_\mu(m^e_j)^2+\delta^j_B\delta^i_\mu(m^e_i)^2)$
15	d	$III\quad VIII$	$g[\mu_P\lambda^{PMn}v_M]\lambda^{iJn}\mu_J(v_A\lambda^{ABj}\mu_B)/(m_{\mathrm{SUSY}}m^e_j)$	$g\delta^i_\mu\delta^j_\mu((m^e_i)^2+(m^e_j)^2)$
16	d	$I\quad III$	$g(v_A\lambda^{ABi}\mu_B)[\mu_P\lambda^{PMn}v_M]\lambda^{jJn}\mu_J/(m_{\mathrm{SUSY}}m^e_i)$	$g\delta^i_\mu\delta^j_\mu((m^e_i)^2+(m^e_j)^2)$

Table 2: Estimated contributions to $[m_\nu]^{ij}$ from all the diagrams. In the second two columns is the label of the diagram of figure 1, and the position on the diagram of the two $\Delta L = 1$ interactions. Column four is the "basis independent" combination of coupling constants for the diagram (which must be symmetrised on $i \leftrightarrow j$), and column five is the estimated contribution to the neutrino mass matrix (in the "flavour basis" where $\langle\tilde\nu\rangle = 0$ and $\hat\nu_i = \vec v\cdot\lambda^i/m^e_i$). All indices other than i and j are summed.